An Expression for Optimum Beamformer SINR in Multipath Channels and Comparison to Adaptive Array Measurements

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Abstract

Optimum beamformer output SINR with one interfering signal is expressed in terms of signal spatial signatures and noise power. The expression, valid for multipath channels, leads to consistent results for the cases of no interference and strong interference. The optimum SINR calculated using the expression is compared to measured SINR achieved using an adaptive beamforming algorithm applied to data recorded using an antenna array testbed.

1. INTRODUCTION

Ishide and Compton [1] derived an expression for SINR in terms of array response vectors for a desired and an interfering signal in the case of a single propagation path for each signal. This expression was used in investigations of conditions that caused grating nulls in adaptive arrays [1] and of suitable element pattern selection to avoid this problem [2]. Subsequent experimental investigations [3]-[5] characterized received signals in terms of spatial signature and measured the variation of spatial signature in dynamic channels. The spatial signature is a complex vector containing the amplitudes and phases of the versions of a signal received by the elements of an antenna array and is applicable to multipath as well as single-path signals. In [6] experimental results were reported and another metric of spatial signature variation was introduced. The spatial signature metrics used in [3]-[6] are reviewed and compared in [7].

In this paper the expression for SINR derived in [1] is written in terms of desired and interfering signal spatial signature vectors. This leads to consistent results for the cases of no interference and strong interference. SINR calculated using the expression is compared to performance of a multi-target least-squares constant modulus algorithm beamformer for data measured using compact four-element antenna arrays.

2. SINR EXPRESSION

Based on [1],[2] the desired signal can be expressed as

$$x_d = A_d U_d s_d(t) \tag{1}$$

where A_d is the signal amplitude, $s_d(t)$ is the normalized signal, and

$$U_d = \begin{bmatrix} f_1(\theta_d,\phi_d,P_d)e^{-j\xi_{d1}} \\ f_2(\theta_d,\phi_d,P_d)e^{-j\xi_{d2}} \\ \vdots \\ f_N(\theta_d,\phi_d,P_d)e^{-j\xi_{dN}} \end{bmatrix} \tag{2}$$

where $f_n(\theta_d,\varphi_d,P_d)$ is the response of the n^{th} element to the desired signal with angle of arrival (θ_d,φ_d) and polarization state P_d and ξ_{dn} is the phase shift at the n^{th} element.

For single-path (e.g., free-space) propagation, we define $V_d = A_d U_d$ *as the spatial* signature, which includes amplitude information. In the general case of a channel with M multipath components, the spatial signature is

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$$V_d = \begin{bmatrix} \sum_{m=1}^{M} \alpha_m f_1\left(\theta_{d_m}, \phi_{d_m}, P_{d_m}\right) e^{-j\xi_{d1m}} \\ \sum_{m=1}^{M} \alpha_m f_2\left(\theta_{d_m}, \phi_{d_m}, P_{d_m}\right) e^{-j\xi_{d2m}} \\ \vdots \\ \sum_{m=1}^{M} \alpha_m f_N\left(\theta_{d_m}, \phi_{d_m}, P_{d_m}\right) e^{-j\xi_{dNm}} \end{bmatrix} = \begin{bmatrix} V_{d_1} \\ V_{d_2} \\ \vdots \\ V_{d_N} \end{bmatrix} \tag{3}$$

and α_{mn} is the amplitude of the m^{th} multipath component received at the n^{th} antenna, $f_n\left(\theta_{d_m}, \phi_{d_m}, P_{d_m}\right)$ is the antenna response to the m^{th} multipath component of the desired signal, and ξ_{dmn} is the phase shift of the m^{th} multipath component at the n^{th} antenna.

We can rewrite equation (18) of [1] to express the output SINR of an optimum beamformer in terms of the spatial signatures as

$$SINR_{opt} = \frac{1}{\sigma^2}\left[V_d^H V_d - \frac{\left|V_d^H V_i\right|^2}{\sigma^2 + V_i^H V_i} \right] \tag{4}$$

where σ^2 is the noise power on one branch. All branches are assumed to have equal noise power and noise on different branches is assumed to be uncorrelated. The SINR can also be written as

$$SINR_{opt} = \frac{1}{\sigma^2}\left[\left\|V_d\right\|^2 - \frac{\left\|V_d\right\|^2 \left\|V_i\right\|^2 \cos^2(\theta_s)}{\sigma^2 + \left\|V_i\right\|^2} \right] \tag{5}$$

where θ_s is the angle between the two spatial signature vectors, mentioned in [3].

The input SNR at the n^{th} element is given by

$$SNR_n = \frac{\left|V_{d_n}\right|^2}{c^2} \tag{6}$$

and the SINR at the n^{th} element is given by

$$SINR_n = \frac{\left|V_{d_n}\right|^2}{\left|V_{i_n}\right|^2 + \sigma^2} \tag{7}$$

where V_{d_n} and V_{i_n} are the n^{th} elements of the desired and interfering signal spatial signature vectors, respectively. For the case of no interference, V_i=0 and the optimum SNR is

$$\begin{aligned} SNR_{opt}\big|_{V_i=0} &= \frac{\left\|V_d\right\|^2}{\sigma^2} \\ &= \sum_{n=1}^{N} SNR_n \end{aligned} \tag{8}$$

which is the optimum result obtained for an ideal maximal ratio combiner [8].

For the case where $\left\|V_i\right\|^2 >> \sigma^2$, the expression for SINR given by equation (5) can be approximated as

$$
\begin{aligned}
SINR_{opt} &\approx \frac{1}{\sigma^2}\left[\|V_d\|^2 - \frac{\|V_d\|^2\|V_i\|^2\cos^2(\theta_s)}{\|V_i\|^2}\right] \\
&= \frac{\|V_d\|^2}{\sigma^2}\sin^2(\theta_s) \\
&= \left(SNR_{opt}\big|_{V_i=0}\right)\sin^2(\theta_s)
\end{aligned}
\qquad (9)
$$

The approximate expression is independent of the magnitude of the interfering signal. The optimum beamformer nulls the interfering signal completely regardless of its spatial signature. If the spatial signature vectors of the two signals are orthogonal, θ_s=90°. In this case the element weights that null the interference are the same as the maximal ratio combining weights. If $V_d = kV_i$, the spatial signature vectors are parallel (θ_s=0°), and a generalized grating null results . This does not necessarily correspond to a null in the array pattern, but the multipath components cancel, both signals are nulled, and $SINR_{opt}$=0.

3. APPLICATION TO MEASURED DATA

The SINR expressions discussed in the previous section can be used to calculate optimum SINR from measured data. Interference rejection measurements were performed using four-element arrays at 2.05 GHz using a four-channel version of the measurement system described in [10]. Two CW signals were transmitted, one from each of two spatially separated transmitters, with a 1kHz offset between the signals. The received signals were mixed to an audio-frequency IF and recorded using two digital audiotape recorders. The recorded digitized signals were then processed on a computer using a multi-target least-squares constant modulus algorithm (MT-LSCMA) beamformer that implements a variation of the algorithm described in [11]. SINR and SNR are calculated for each signal before and after beamforming. Figure 1 shows the spatial signature angle in degrees (θ_s) between two measured signals, Fig. 2 shows the difference between optimum SINR (5) and optimum SNR in the non-interference case (8), and Fig. 3 shows SINR calculated using (5) and measured SINR of the LSCMA beamformer for data recorded using a handheld array in a microcell interference scenario. Agreement between the spatial signature variation, calculated SINR, and measured SINR is especially evident in the peaks at 1.44, 1.68, 1.8 and 1.92 seconds, and in the nulls at 1.75, 1.86, and 1.97 seconds.

4. CONCLUSION

SINR is expressed in terms of signal spatial signatures. Results agree well with measured SINR using an LSCMA beamformer on data recorded using an array testbed.

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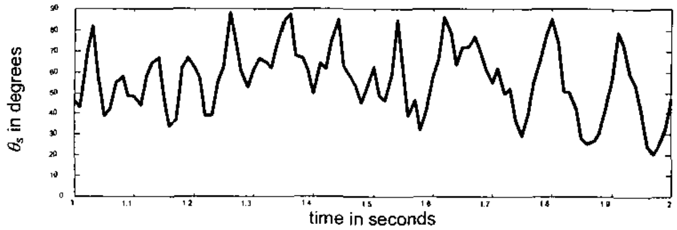

Figure 1 *Spatial signature angle θ_s as a function of time for measured signals*

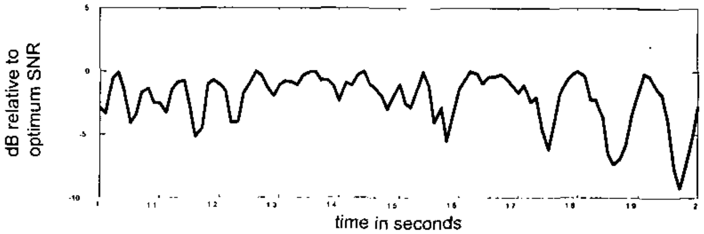

Figure 2 Optimum SINR relative to optimum SNR in non-interference case

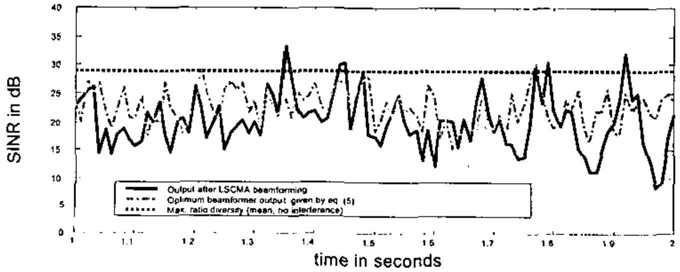

Figure 3 Calculated optimum SINR, measured SINR for LSCMA, and optimum SNR